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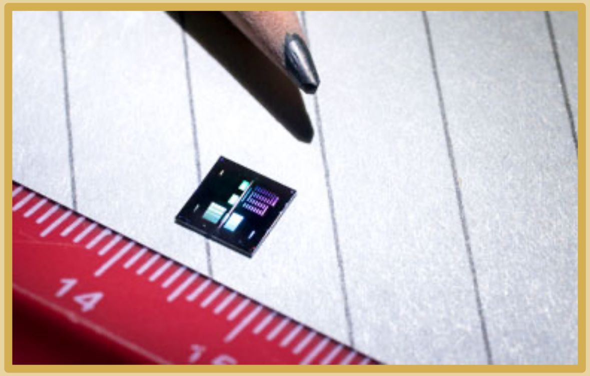
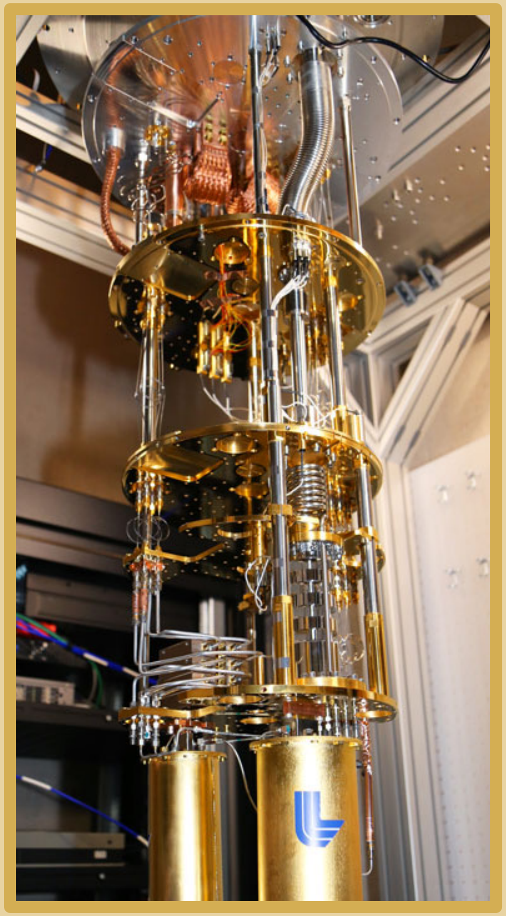
Bilinear Dynamic Mode Decomposition for Quantum Control

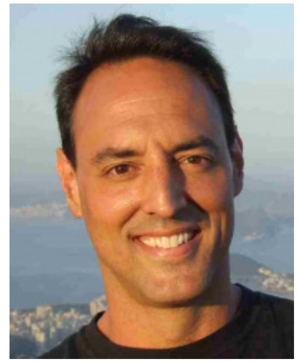
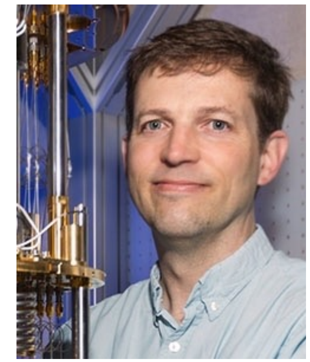


< Quantum chip
for transmon qubits



^ Dilution refrigerator

Acknowledgements

Steve Brunton, Eurika Kaiser, & Nathan Kutz

Jonathan DuBois

UNIVERSITY *of* WASHINGTON

Lawrence Livermore National Laboratory

QISE-NET

QUANTUM INFORMATION SCIENCE AND ENGINEERING NETWORK

Goals

> Data-driven discovery of effective Hamiltonian models
> ...For the optimal control of quantum gates and common algorithm sub-routines
> ....To enhance the utility of Noisy, Intermediate Scale Quantum (NISQ) devices

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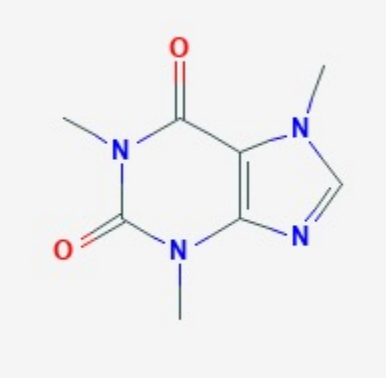
Building a quantum computer

> **How many qubits do we want?**



Caffeine ground state:
100s of qubits

> **1000s of high-fidelity (99.9%) physical qubits for 1 logical qubit using quantum error correction.**

> **NISQ era: 100-1000s of physical qubits without error correction**

J. Preskill, Quantum. 2, 79 (2018).

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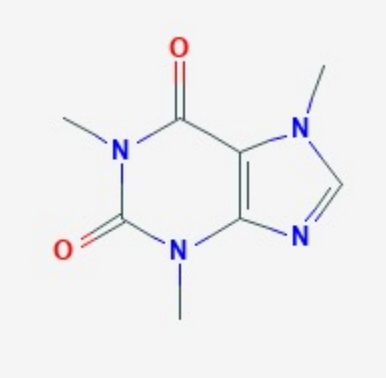

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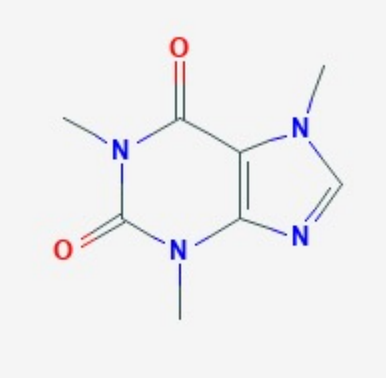

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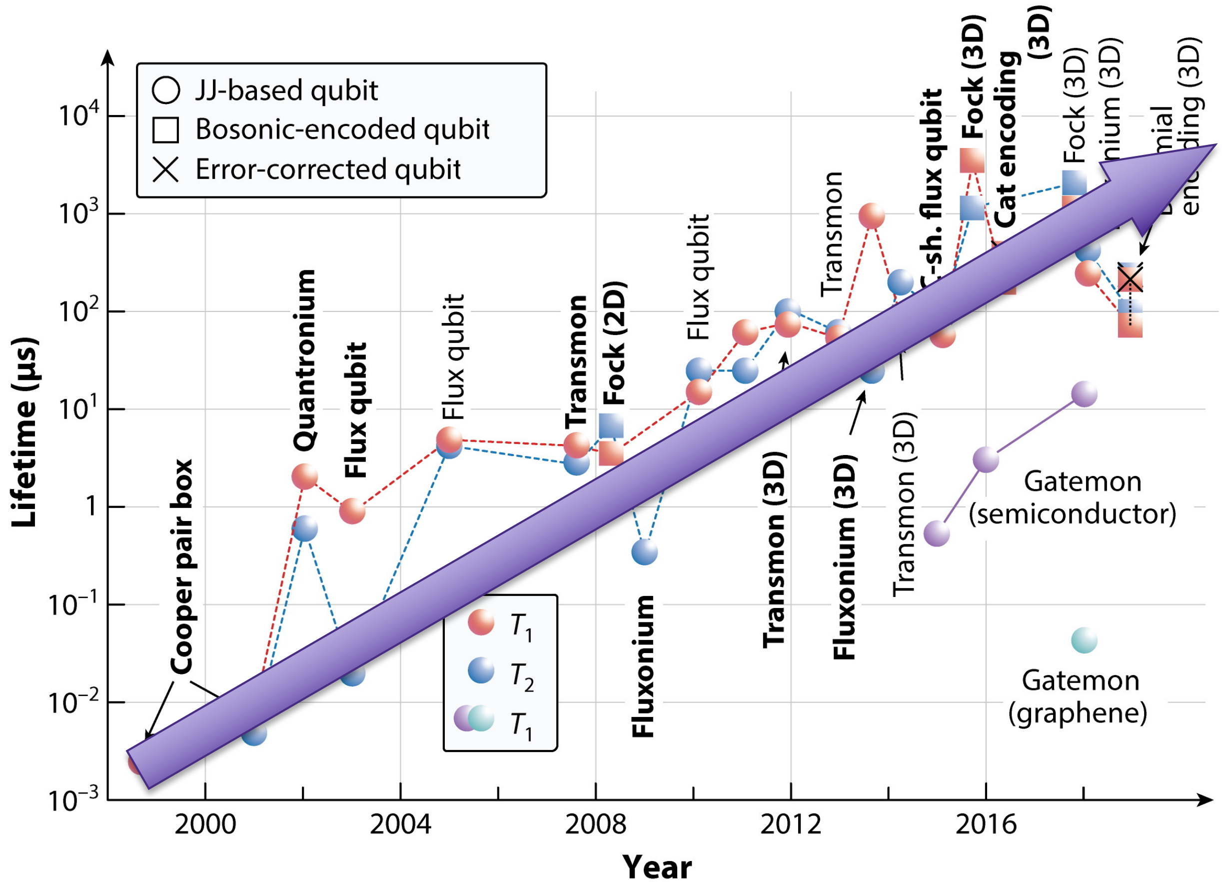
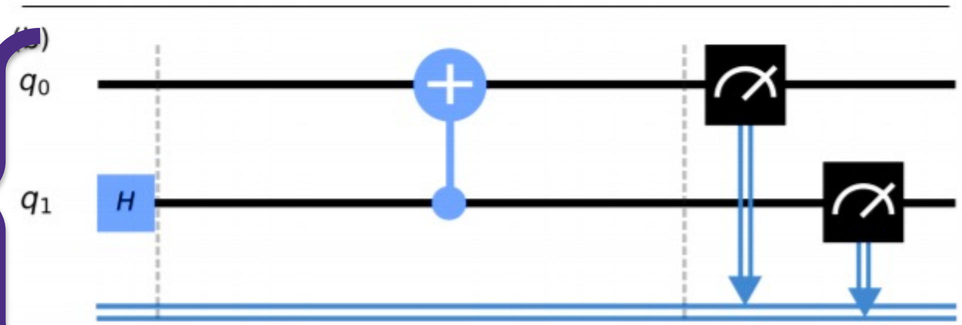
Moore's law for superconducting qubits



M. Kjaergaard et al. Annu. Rev. Condens. Matter Phys. 11, 369–395 (2020)

Quantum computing: Layers of abstraction

Circuit model
(2 qubits)

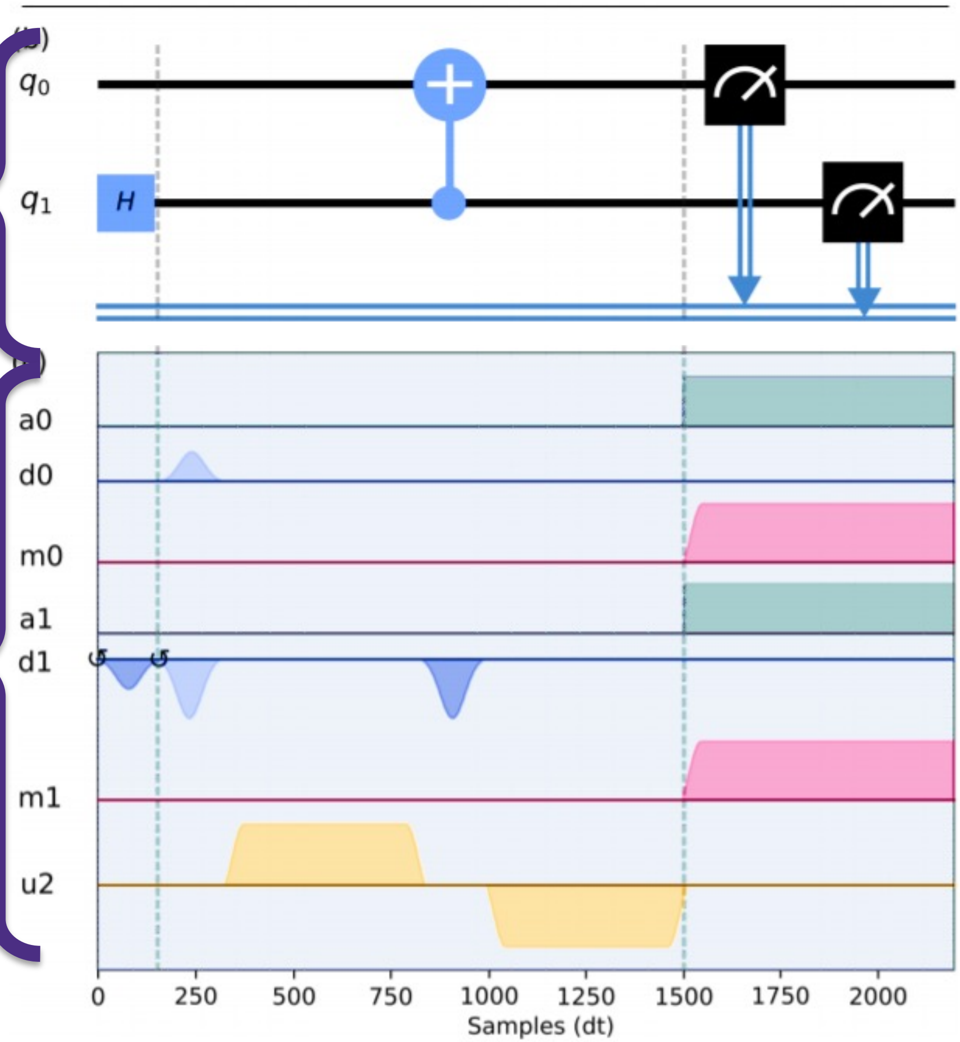


Thomas Alexander et al. Quantum Sci. Technol. **5** 044006 (2020)

Quantum computing: Layers of abstraction

Circuit model (2 qubits)

Pulse sequences for control and measurement



Thomas Alexander et al. Quantum Sci. Technol. **5** 044006 (2020)

Pulse-level control

- Quantum dynamics act linearly on the state.

$$\frac{\partial \Psi}{\partial t} = -iH\Psi$$

- Quantum control dynamics are bilinear in the state and control.

$$\frac{\partial \Psi}{\partial t} = -iH(u)\Psi$$

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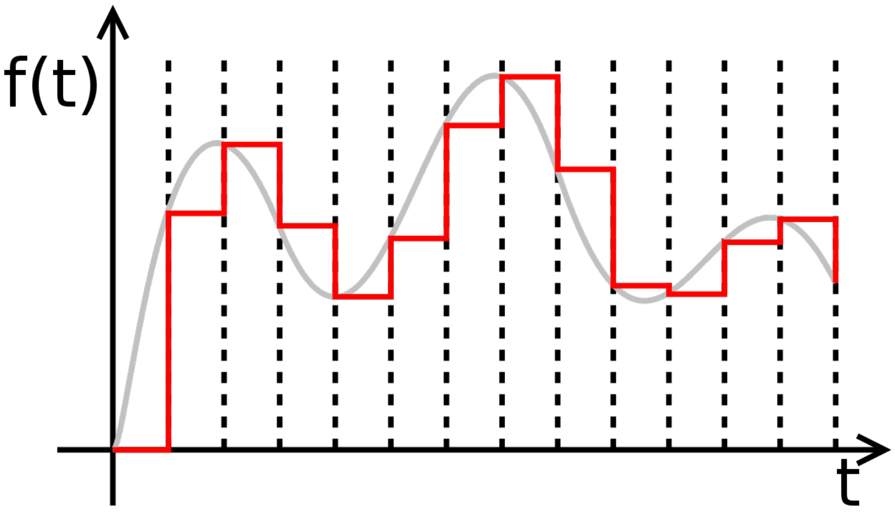
Optimal control



> **Open loop: Design pulse using known models.**

> Optimize a cost function to improve upon parameterized pulses.
  - GRAPE, CRAB, Krotov, etc.

> Bad models mean bad gates.

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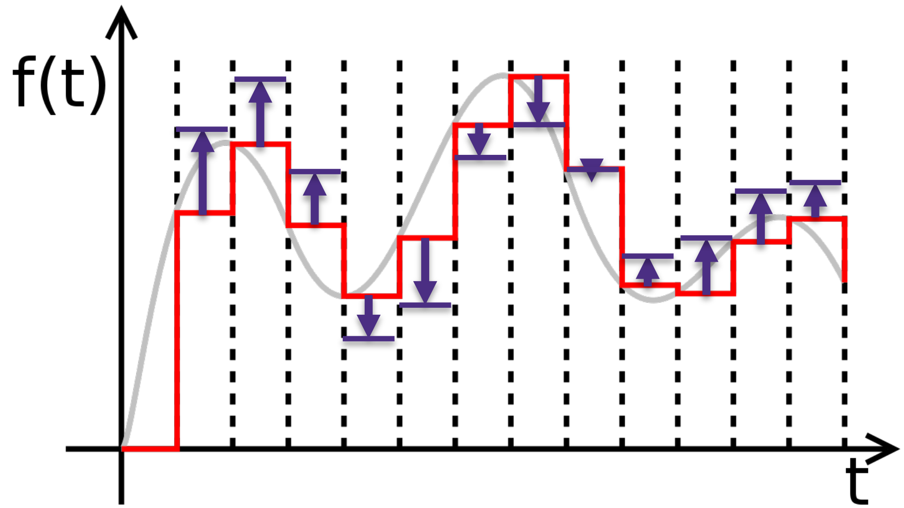

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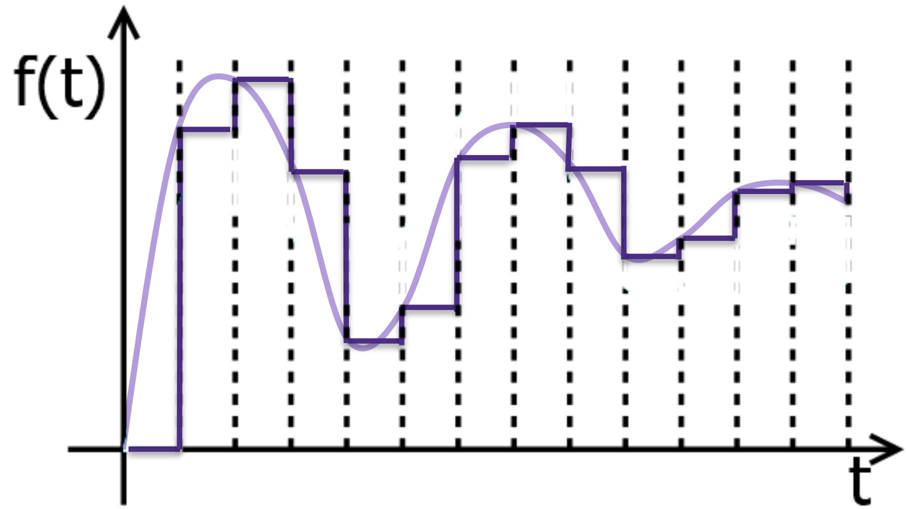

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Modeling challenges

1. Unknown dynamics
2. Noise
3. Leakage to non-qubit states
4. Nonlinear control distortions
5. Cross-talk

> Difficulty increases with scale.
  - A balancing act: protect (detune) vs. operate (couple)

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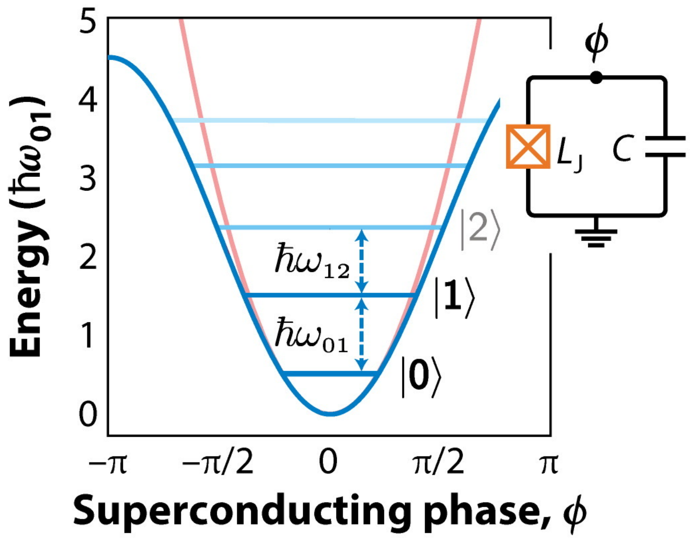

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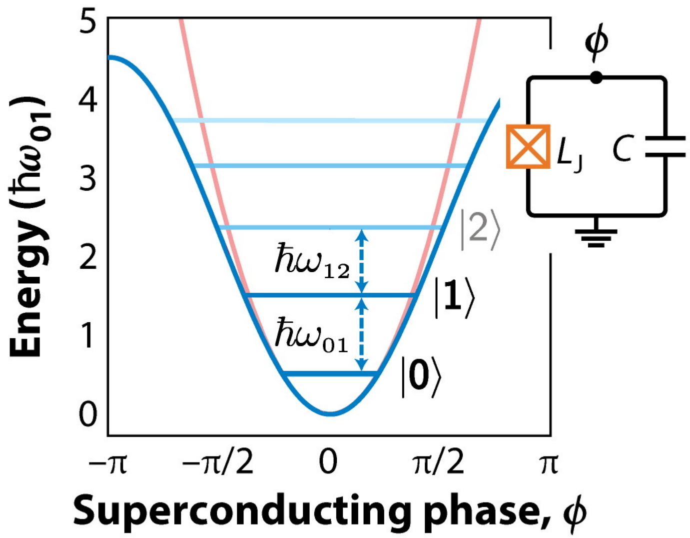

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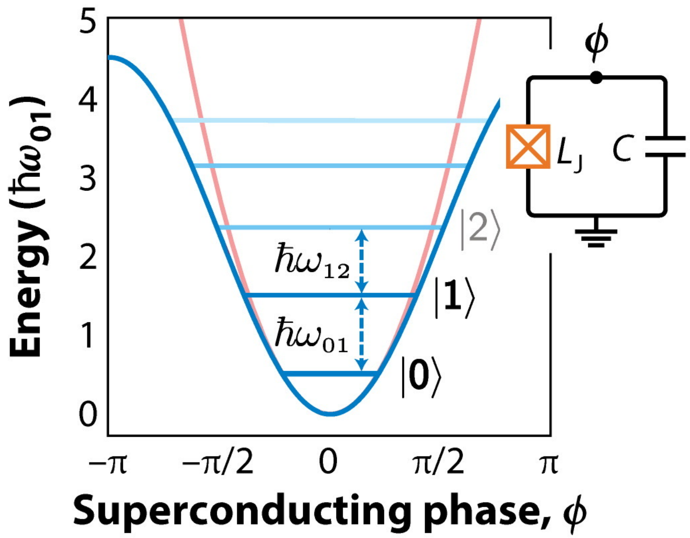
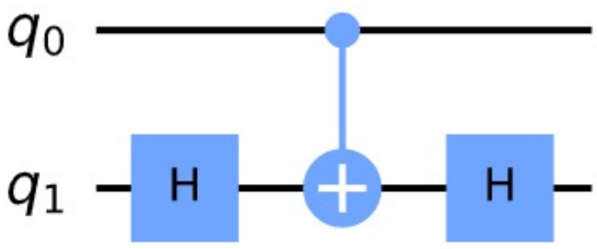


Example: CZ gate



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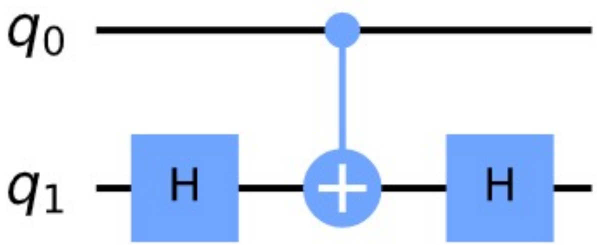
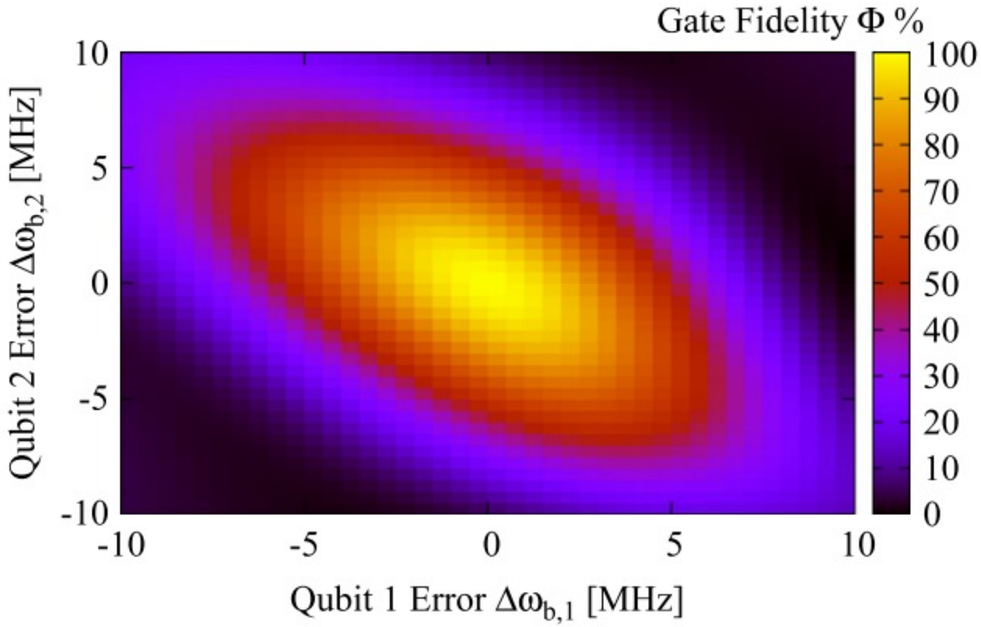


> Even modeling errors $\Delta < 1\%$ ruined the gate.



D J Egger and F K Wilhelm 2014 *Supercond. Sci. Technol.* **27** 014001

Improvements

Use data.

> **Model-based**

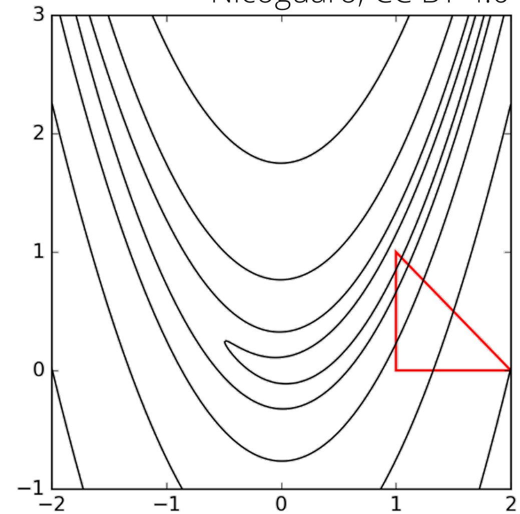
> **Model-free**

- ***Ex post facto*** **pulses "suited to the single yet uncertain physical system at hand".**

  D. J. Egger, F. K. Wilhelm,. Phys. Rev. Lett. 112 (2014)

- **Reinforcement learning**

> **Synthesis: Data-driven models suited to the system at hand.**



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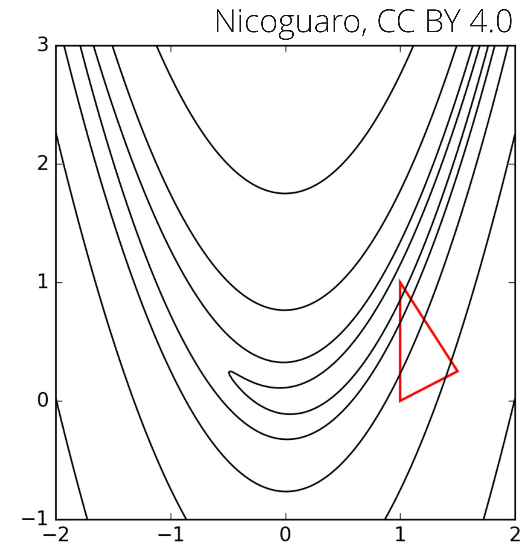

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Dynamic mode decomposition

Can we retain the underlying (effective) Hamiltonian structure?

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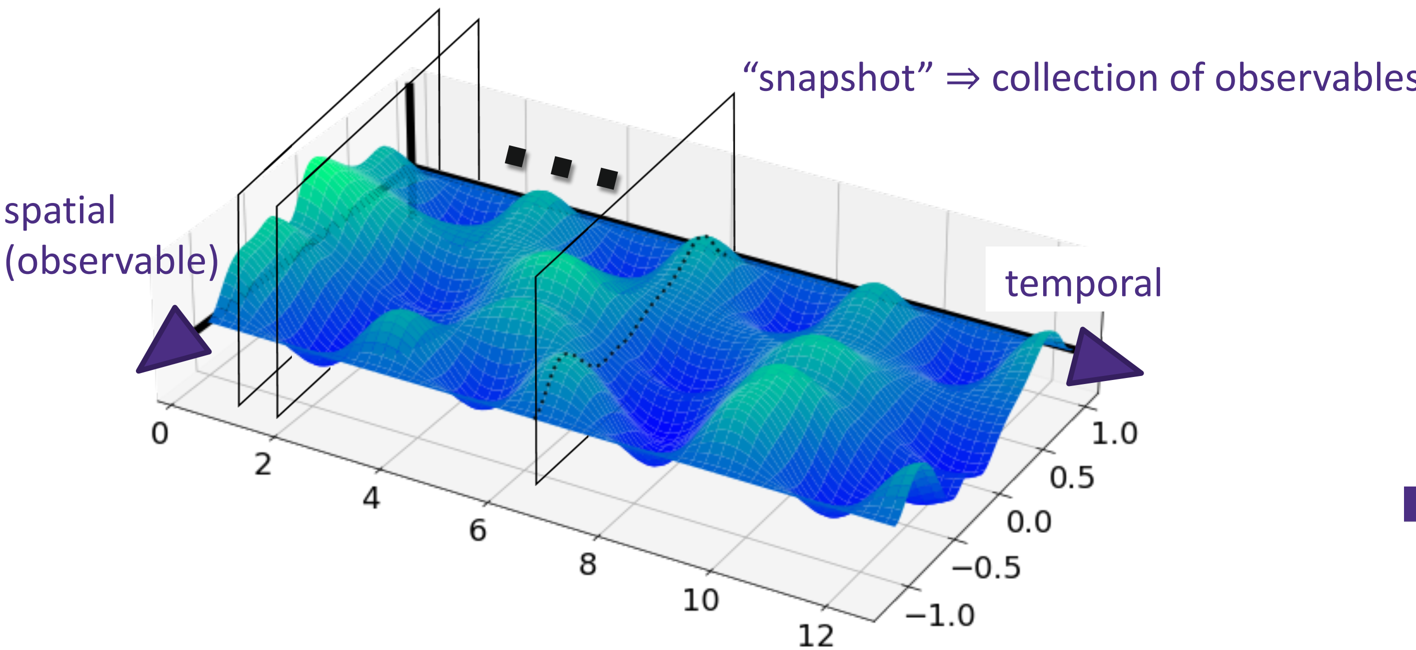
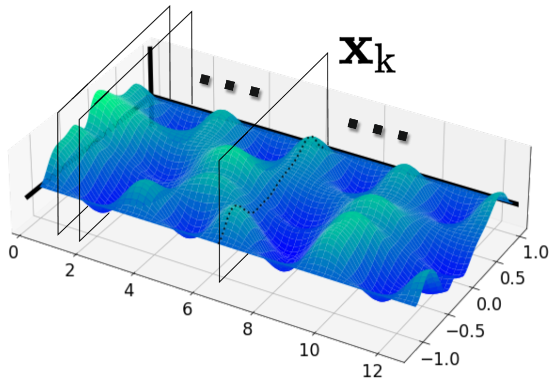
> **Goal: find a generator that describes the dynamics of a collection of observables.**



DMD: model-free regression.

1. Collect data.



2. Assemble snapshot matrices.

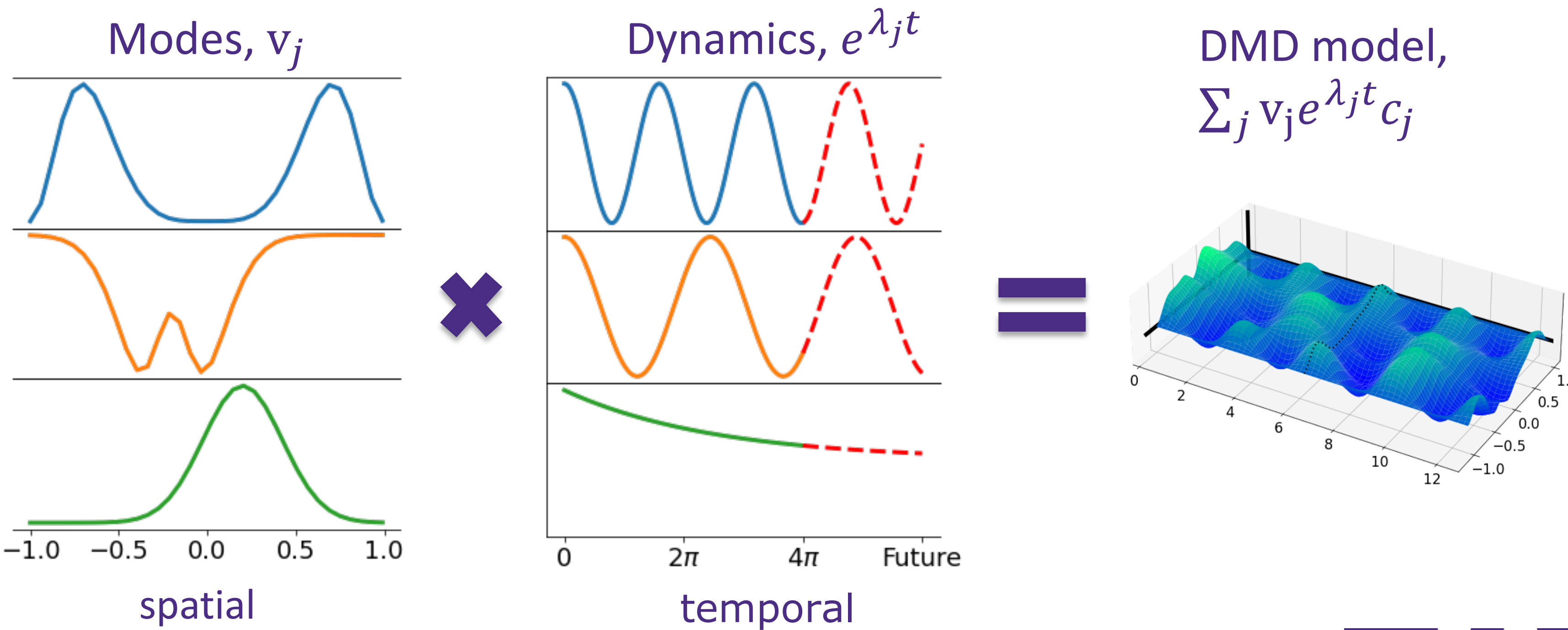
$$\mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{x}_1 & \mathbf{x}_2 & \cdots & \mathbf{x}_{M-1} \\ | & | & & | \end{bmatrix}$$

$$\mathbf{X}' = \begin{bmatrix} | & | & & | \\ \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_{M} \\ | & | & & | \end{bmatrix}$$

3. Compute the regression.

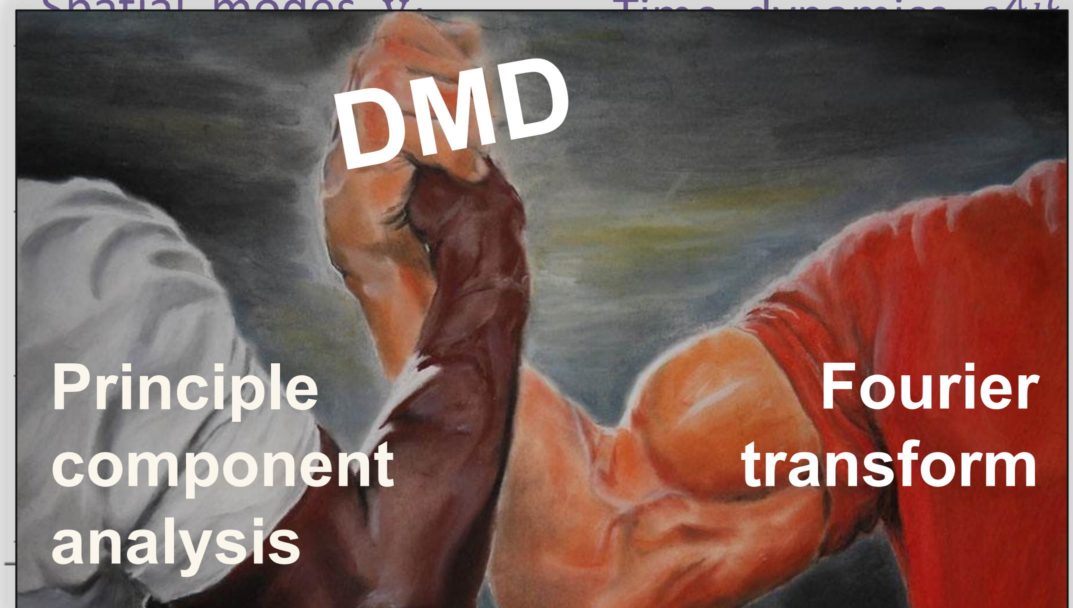
$$\mathbf{X}' \approx \mathbf{A}\mathbf{X} \quad \Rightarrow \quad \mathbf{A} \leftarrow \mathbf{X}'\mathbf{X}^{+}$$

DMD: model reduction.



DMD: model reduction.

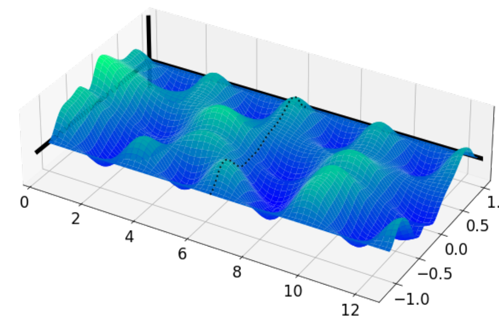
Spatial modes v_j Time dynamics $e^{\lambda_j t}$



spatial temporal

=

DMD model,
$\sum_j \mathrm{v_j} e^{\lambda_j t} c_j$



Quantum system identification

Examples:

- **Hamiltonian identification**
  - **Via special-case quantum process tomography**

    E.g., Y. Wang, et al. IEEE Trans. Automat. Contr. 63, 1388–1403 (2018).
- **Measurement time traces**
  - **Eigensystem realization algorithm**

    J. Zhang, M. Sarovar, Phys. Rev. Lett. 113 (2014)
- **We want disambiguation of the dynamics from the effects of actuation.**

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> **We want disambiguation of the dynamics from the effects of actuation.**

Bilinear dynamic mode decomposition

Modifying DMDc for quantum control

J. Proctor et al. SIAM J. Appl. Dyn. Syst. (2016).

> **Quantum control dynamics are bilinear:**

$$\boldsymbol{H}(\boldsymbol{t}) = \boldsymbol{H}_{\mathbf{0}} + \sum_{j=1}^{J} \boldsymbol{u}_j(\boldsymbol{t})\boldsymbol{H}_j$$

> **In terms of snapshot matrices, we want:**

$$\mathbf{X}' \approx \mathbf{A}\,\mathbf{X} + \mathbf{B}\,(\mathbf{U} * \mathbf{X})$$

A. Goldschmidt et al. New J. Phys. 23 033035 (2021).

S. Peitz et al. SIAM J. Appl. Dyn. Syst. 19, 2162–2193 (2020).

I. Gosea, I. Duff. arXiv:2003.06484 [math.NA] (2020).

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3. Multiply.

$$\mathbf{U} * \mathbf{X} = \begin{bmatrix} | & | & & | \\ \mathbf{u}_1 \otimes \mathbf{x}_1 & \mathbf{u}_2 \otimes \mathbf{x}_2 & \dots & \mathbf{u}_{M-1} \otimes \mathbf{x}_{M-1} \\ | & | & & | \end{bmatrix}$$

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Using biDMD for quantum control

> Extend biDMD with respect to the control.

$$\mathbf{u}_\mathrm{k} \mapsto \begin{bmatrix} g_1(\mathbf{u}_\mathrm{k}) \\ g_2(\mathbf{u}_\mathrm{k}) \\ \vdots \\ g_b(\mathbf{u}_\mathrm{k}) \end{bmatrix}$$

M. O. Williams et al., J Nonlinear Sci. 25, 1307–1346 (2015).

- Effective Hamiltonians produce lifted controls.
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Two Examples

Apply bilinear DMD to a qubit in a rotating frame.

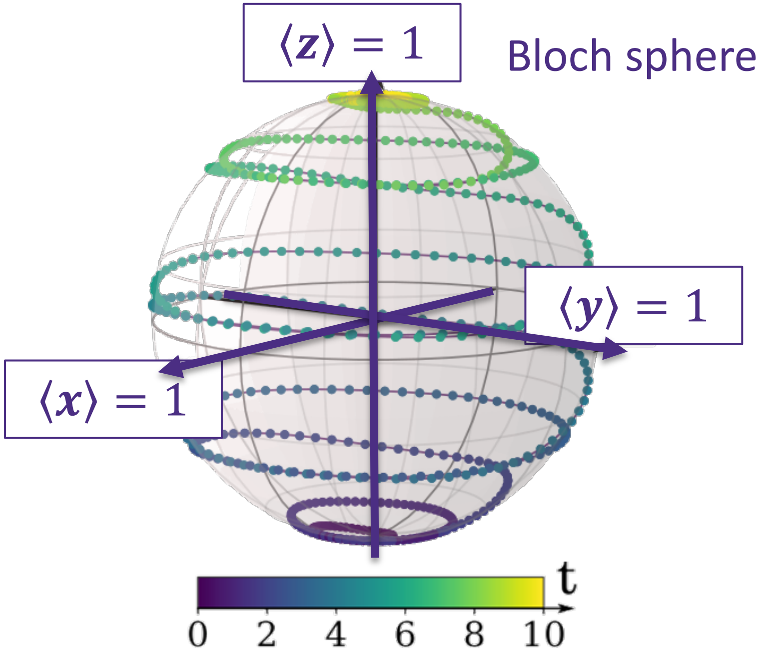
> Physical model:

$$\mathbf{H}(\mathrm{t}) = \Delta\, \boldsymbol{H}_{\mathbf{0}} + u(t)\, \boldsymbol{H}_{\mathbf{1}}$$

> Snapshots:

State: $\mathbf{x}(\mathrm{t}) = \begin{bmatrix} \langle \boldsymbol{x}\,(t) \rangle \\ \langle \boldsymbol{y}(t) \rangle \\ \langle \boldsymbol{z}(t) \rangle \end{bmatrix}$

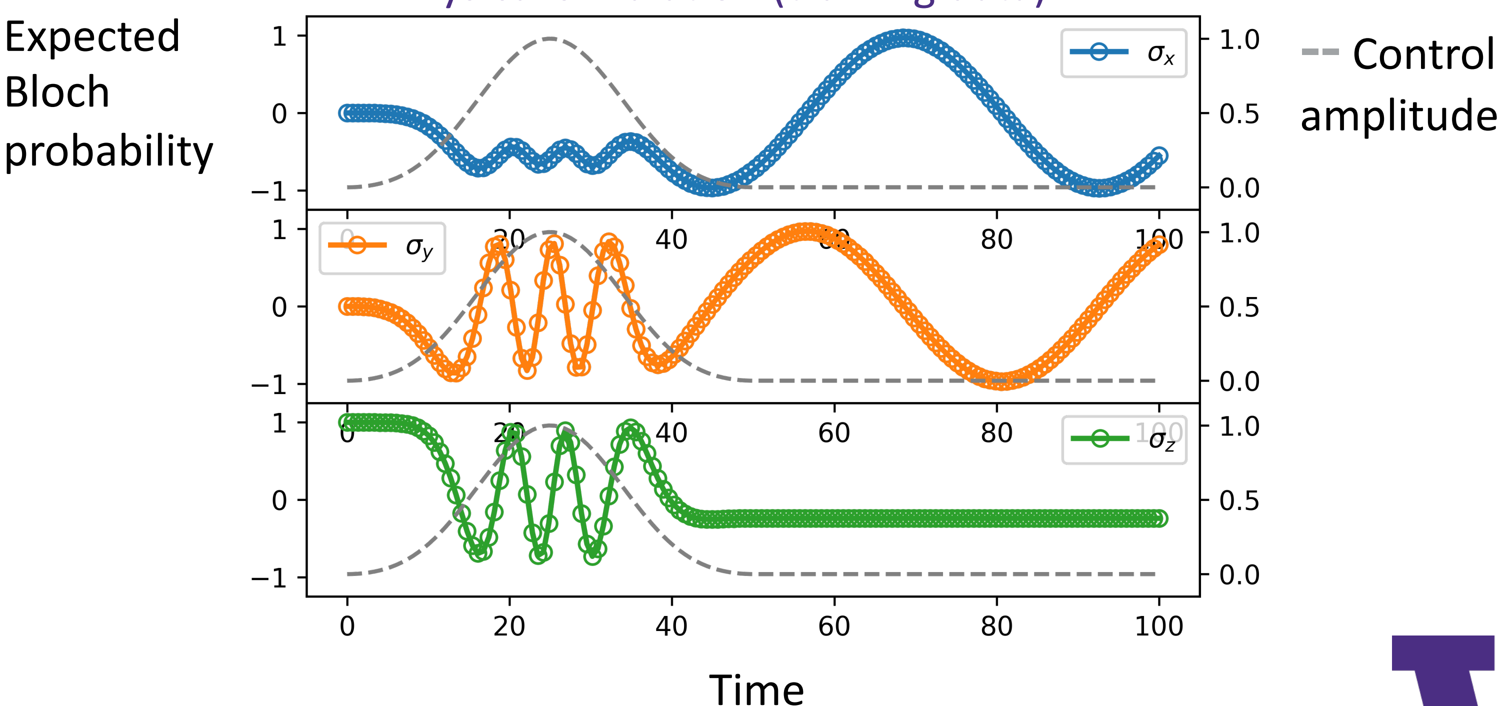
Control: $\mathbf{g}(u(t)) = \begin{bmatrix} u(t) \\ u(t)^2 \\ u(t)^3 \\ u(t)^4 \end{bmatrix}$



Example 1

> Identify an effective model for a qubit.

Physical simulation (training data)



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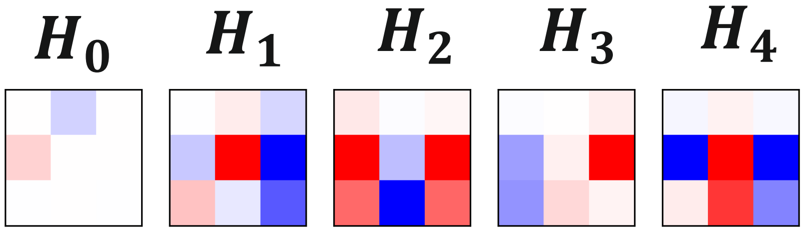

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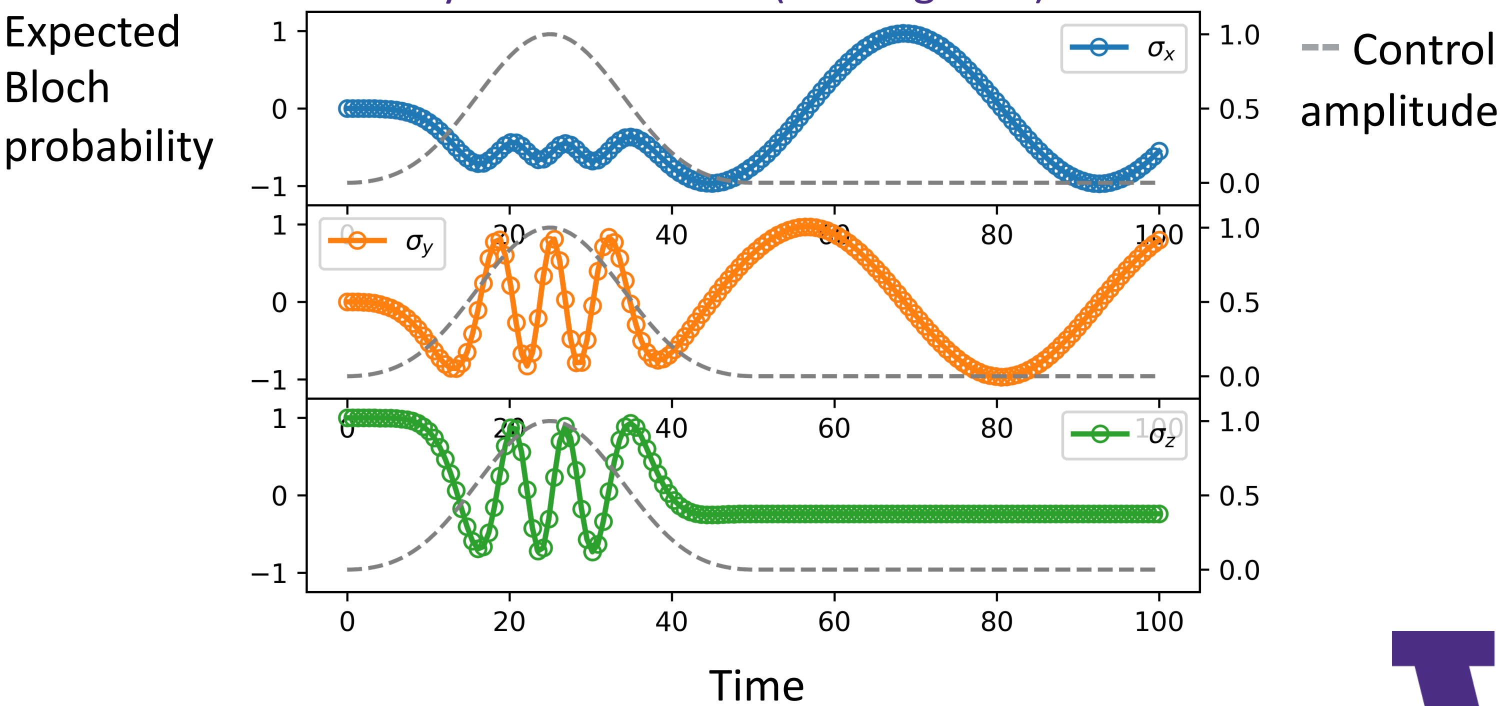

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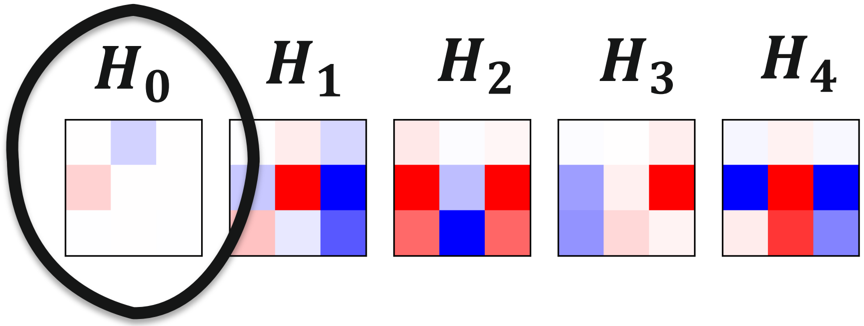
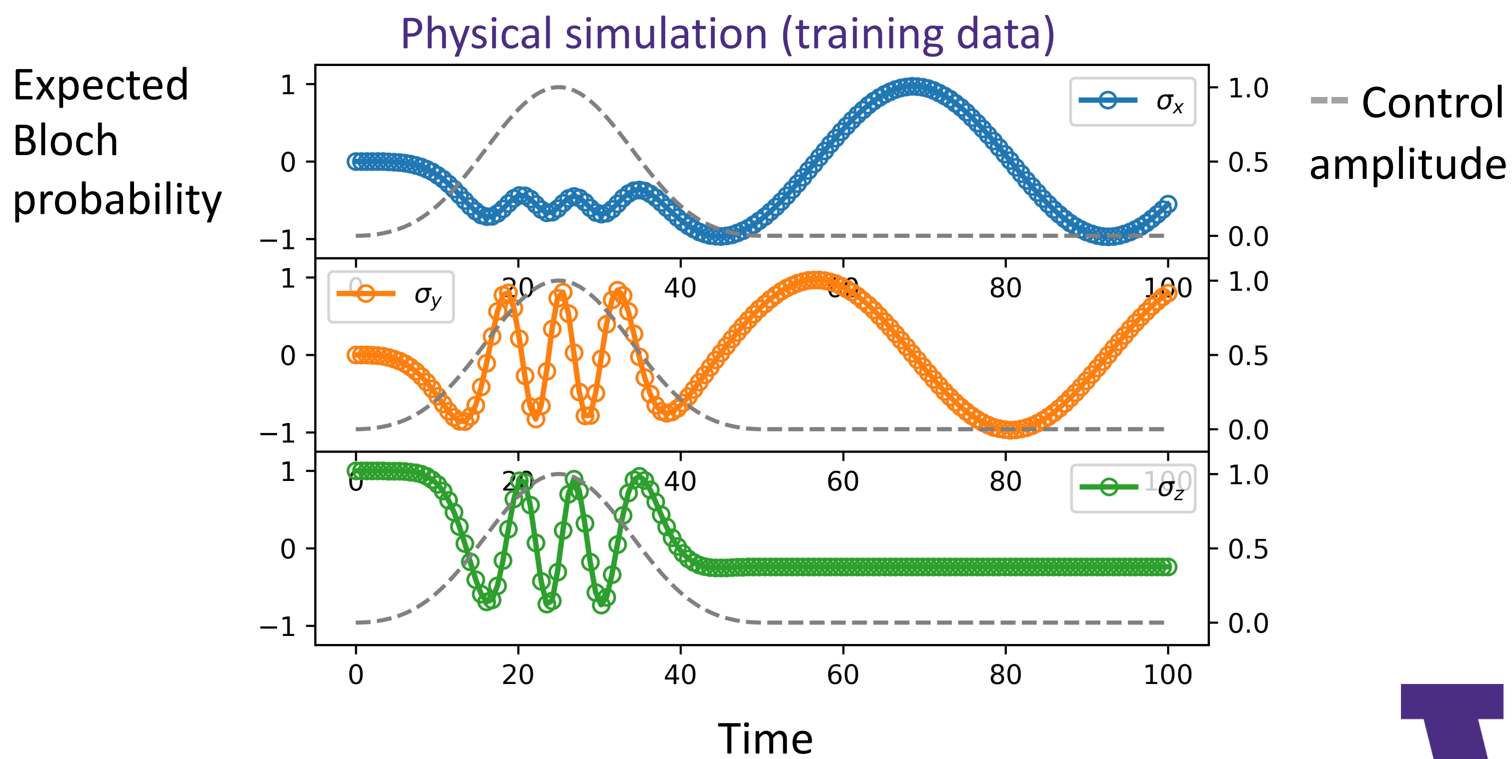
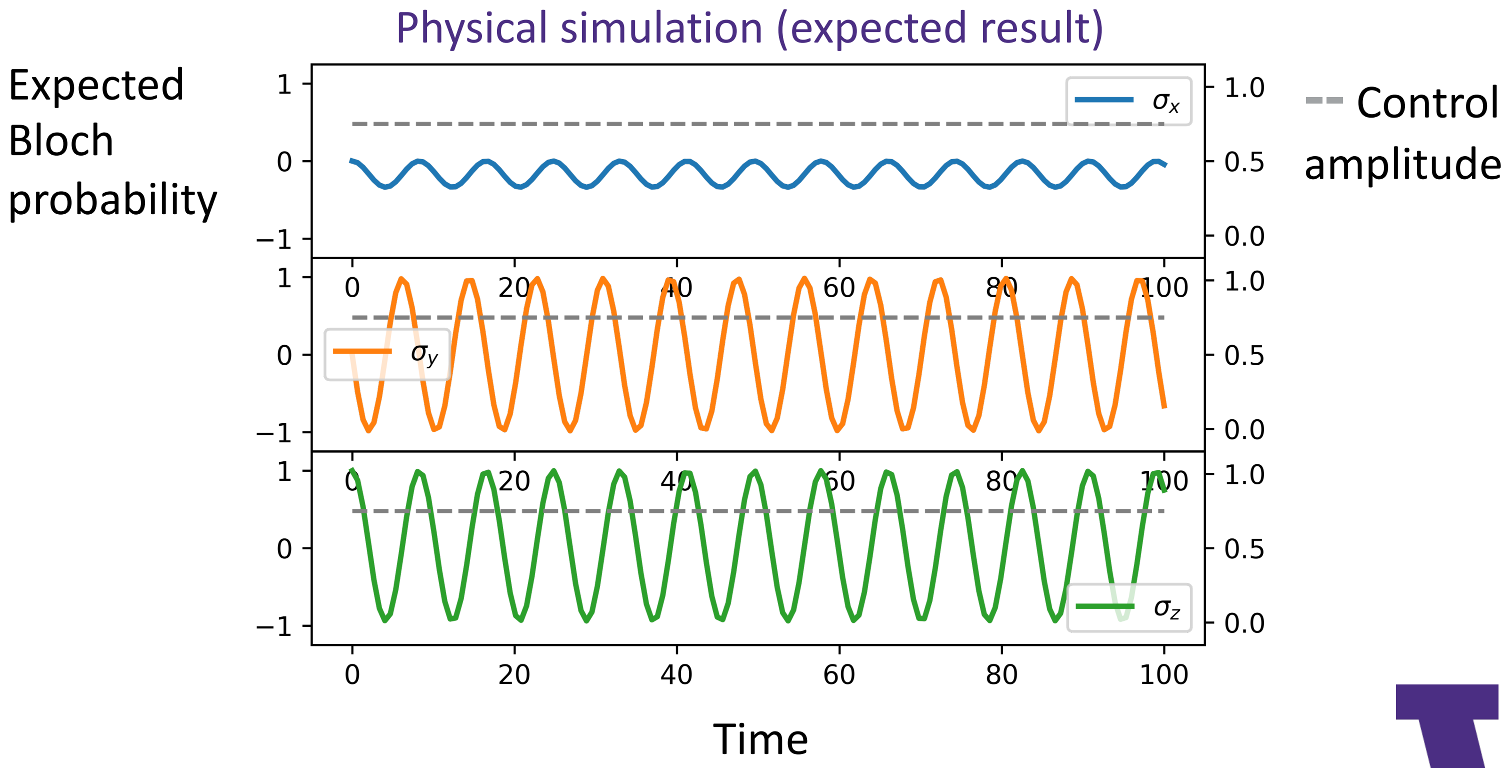


> Identify an effective model for a qubit.



Example 1

> Test by playing a constant pulse.

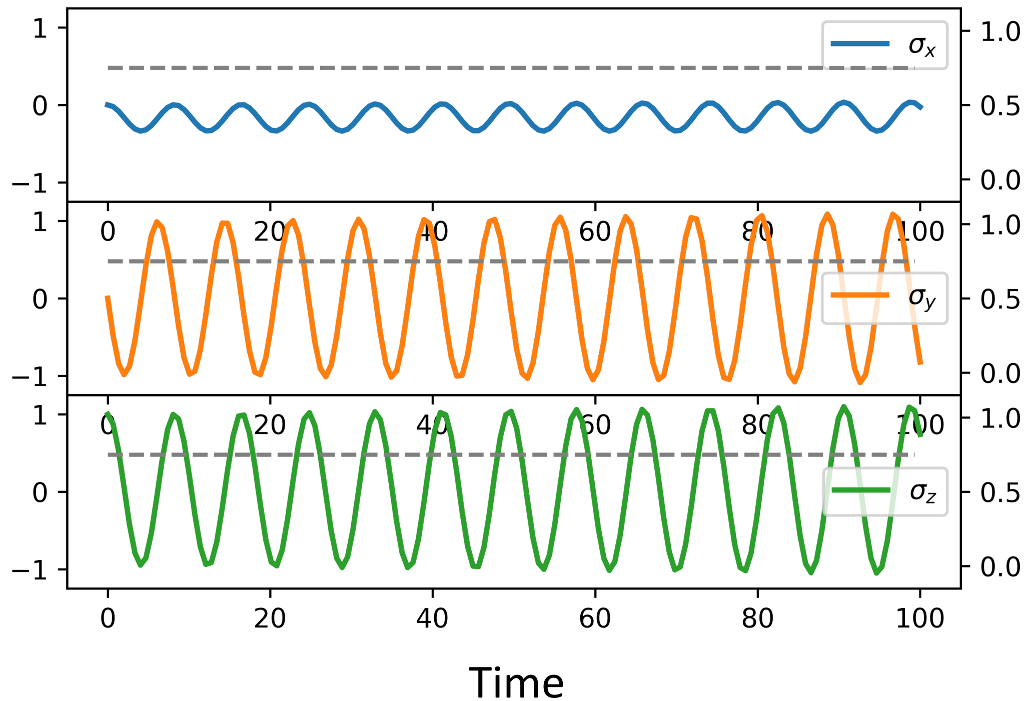


Example 1

> The biDMD accurately predicts the dynamics with just the initial condition and the new control.

Model prediction

Expected Bloch probability



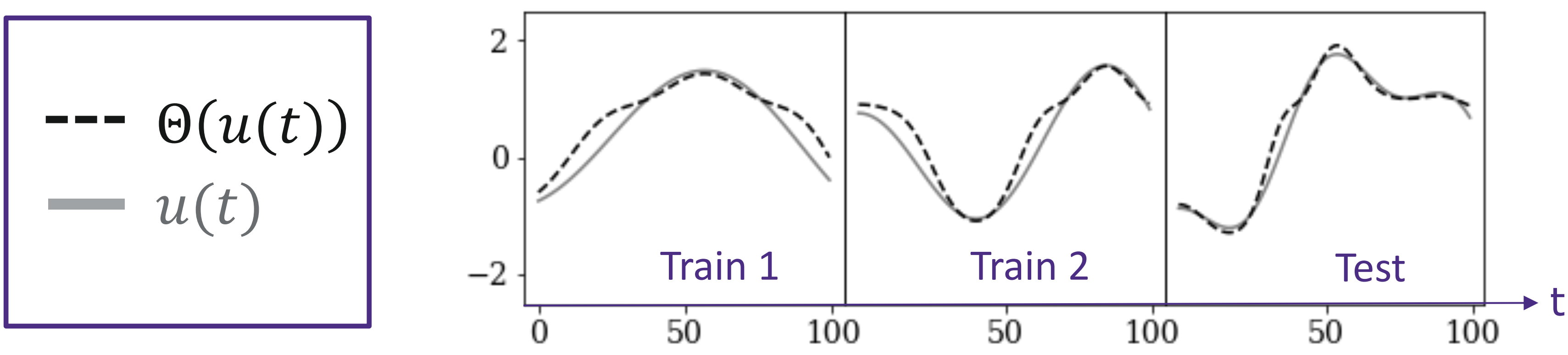
-- Control amplitude

Subtle increase > 1, not physical.

Example 2

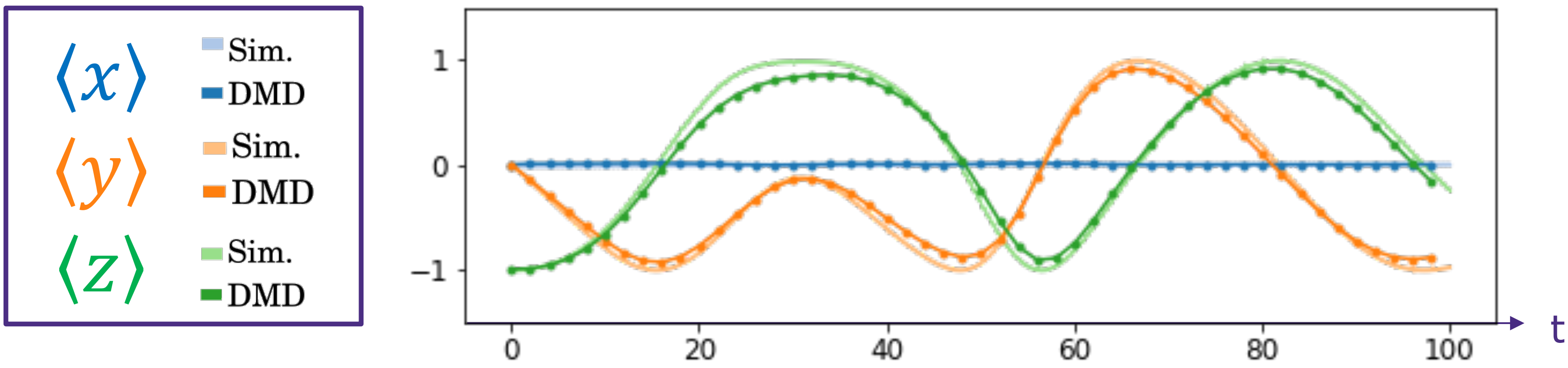
> Identify an effective model for a qubit in the presence of unknown nonlinear distortions on the control:

$$\Theta(u) = u + 0.1u^2 + \cos(u)$$



Example 2

> A biDMD model with the extended control basis, $\mathrm{g}(u)$, captures the nonlinearity in the test pulse.



Takeaways

> There are lots of ways for modeling to go wrong.

> Separately isolating each error can be difficult.

> In a single framework, biDMD accommodates the unknown dynamics in an interpretable way.

BiDMD

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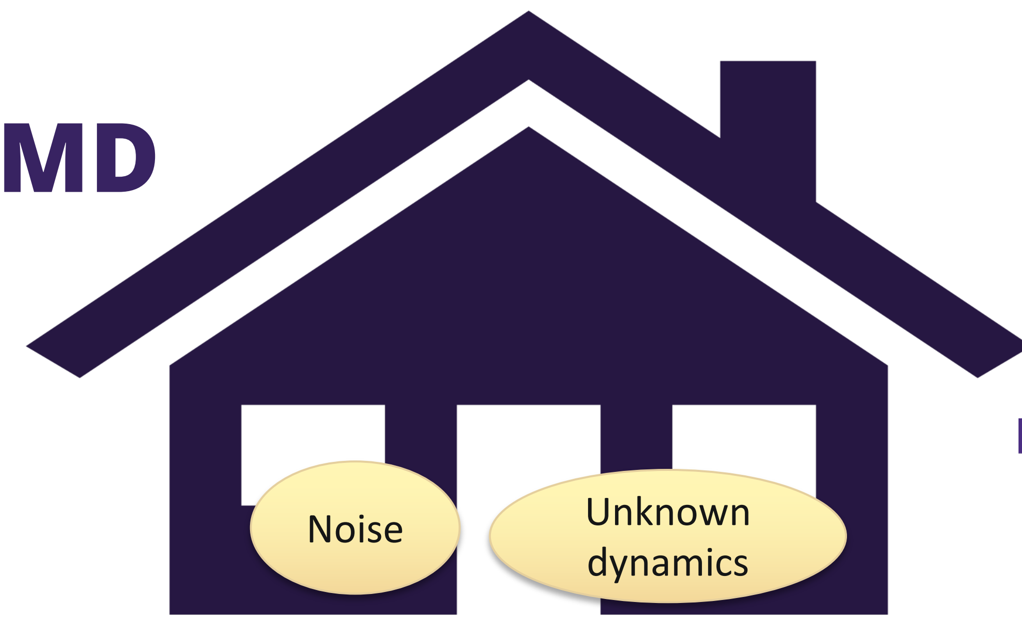

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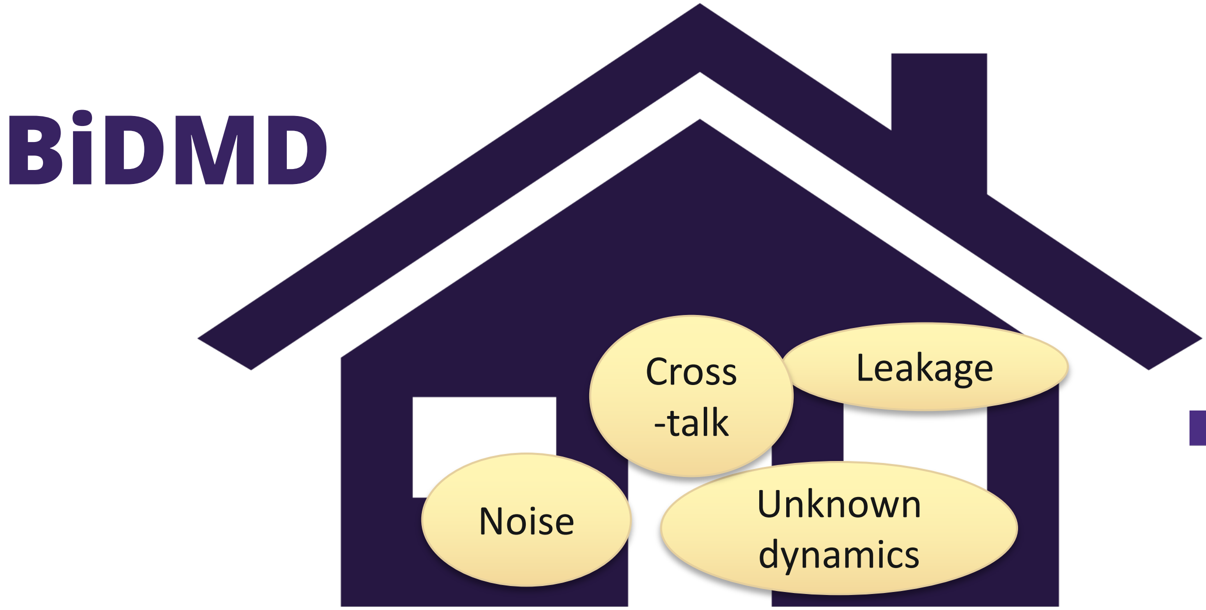

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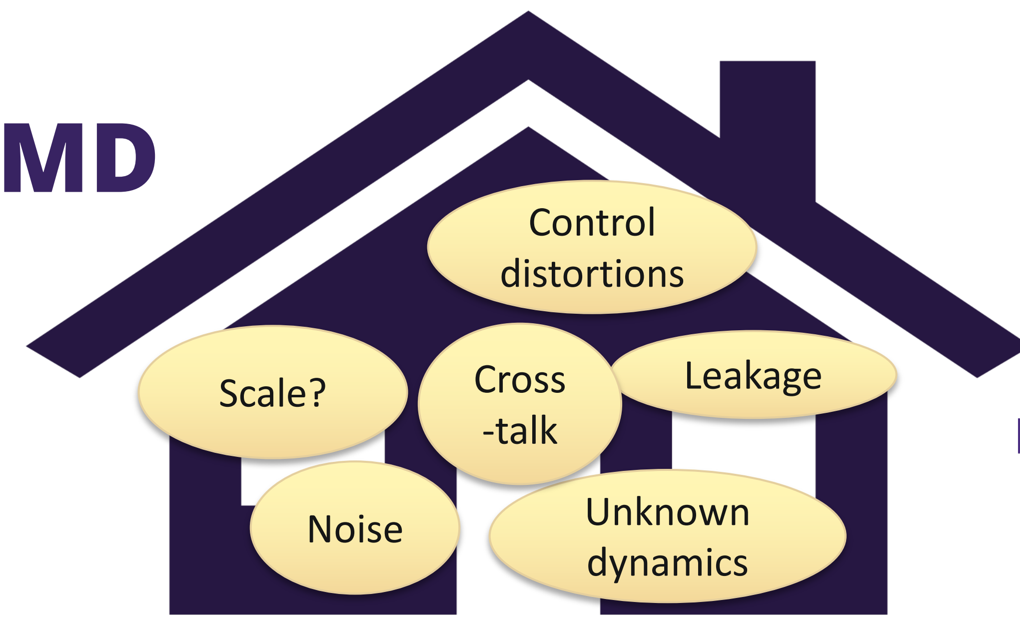
BiDMD



Summary + questions?

A. Goldschmidt, E. Kaiser, J. L. DuBois, S. L. Brunton, J. N. Kutz, *Bilinear dynamic mode decomposition for quantum control*. New J. Phys. 23, 033035 (2021).

New work on model predictive control coming soon!

github.com/andgoldschmidt

UNIVERSITY of WASHINGTON

Backup slides

Extrapolation error for DMD

Model discrepancy at step n from training size m,

$$\|\mathbf{x}_\mathrm{n} - \hat{\mathbf{x}}_\mathrm{n}\|_2 \leq \kappa\left(\|\mathbf{x}_\mathrm{m} - \hat{\mathbf{x}}_\mathrm{m}\|_2 + (n-m)\varepsilon_m\right)$$

Training fit

Accumulation from numerical integration

- True dynamics, $\mathbf{x}$
- Predicted dynamics (only $\mathbf{x_0}$ known), $\hat{\mathbf{x}}$
- Number of DMD modes, κ
- In the limit of more snapshots, $\varepsilon_m \to 0$

Outside the RWA

- In most quantum examples, the rotating wave approximation is appropriate.
- We also looked at the case where we cannot make this approximation.
  - DMD can accommodate *stroboscopic* data using the *Floquet theory* and the *Magnus expansion* (see our paper for more).

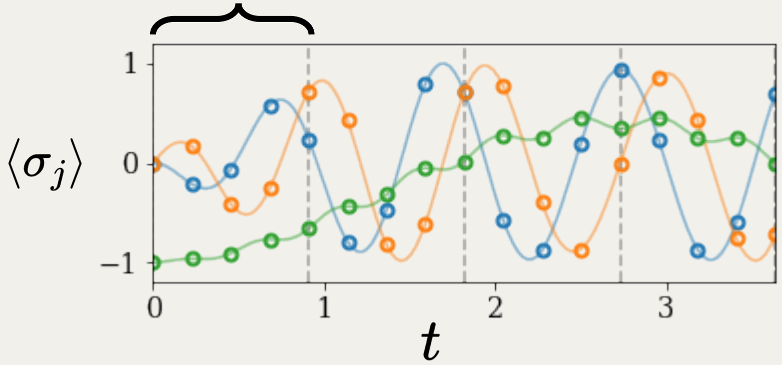
Example 3 (Floquet DMD)

$$u(t) = u_0 \cos(\omega t)$$

Like Example 1, drive a strongly-coupled qubit slightly off-resonance.

Collect snapshots into a Floquet data matrix with T-periodic columns.

$$\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_s$$



$$\mathbf{X}_\mathrm{F} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_{s+1} & \cdots & \mathbf{x}_{(m-1)s+1} \\ \mathbf{x}_2 & \mathbf{x}_{s+2} & \cdots & \mathbf{x}_{(m-1)s+2} \\ \vdots & \vdots & & \vdots \\ \mathbf{x}_s & \mathbf{x}_{2s} & \cdots & \mathbf{x}_{(m-1)s+s} \end{bmatrix}$$

$$\mathbf{X}'_\mathrm{F} = \begin{bmatrix} \mathbf{x}_{s+1} & \mathbf{x}_{2s+1} & \cdots & \mathbf{x}_{ms+1} \\ \mathbf{x}_{s+2} & \mathbf{x}_{2s+2} & \cdots & \mathbf{x}_{ms+2} \\ \vdots & \vdots & & \vdots \\ \mathbf{x}_{2s} & \mathbf{x}_{3s} & \cdots & \mathbf{x}_{ms+s} \end{bmatrix}$$

Floquet DMD resolves the fast scale dynamics.

$$\mathbf{x}(t) = \sum_j \boldsymbol{\xi}_j(t) e^{\varepsilon_j (t-t_0)} c_j$$